Multilateral Analysis of PET bottle oscillator

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Abstract

An upside-down bottle containing water repeats the outflow of water and inflow of air. We connected a tube to the head of the plastic bottle, and called this equipment a PET bottle oscillator. It is used as an easy model to analyze oscillating objects. Besides, we connected the bottom of two PET bottle oscillators with the tube, and we saw the rhythm of them becoming same. We call this phenomenon synchronization. In our research, we focused on the synchronization, analyzed its oscillating periods, and tried to express the periodic movements of the oscillating objects by using mathematical formulas. Finally, we succeeded in making mathematical models for the synchronization of the frequency and phases.

1. Purpose

When things which have the same characteristics interact with each other, they establish the order as a whole. This phenomenon is called synchronization and we can see it everywhere in our daily lives. The glows of fireflies and the rhythm of a heart have also synchronization. However, to analyze these biological or chemical synchronizations is difficult because in these synchronizations, each aspect has a complex influence on each other. So, to understand these phenomena, it is important to analyze physical synchronizations which can be easily conducted. Therefore, we focused on the synchronization of the PET bottle oscillator, the reverse phase, and the phenomenon in which the difference in the phase of frequency becomes π. The inflow of air in the PET bottle oscillator is periodic. Therefore, it is one of the popular targets to study in Japan. Our senior student in physics club found that the period of one PET bottle oscillator became longer and made the high quality method of measuring the force from the PET bottle oscillator accurately. Also in 2005 Mr. Tajima conducted an experiment to measure the volume of air and inner pressure. In 2006 Mr. Kodaira made mathematical models determined whether water flows or not. Mr. Kikuchi also made mathematical models by using the flowing speed of water and the inner pressure as parameters. In 2011 a group of Setoyama created mathematical models in terms of the radius and length of a tube. This time, we focused on the state of the phase of the PET bottle oscillator and made mathematical model taking Kuramoto Model into account.

2. Method

2-1 Laboratory instrument

Two bottles of PET bottle (volume: 2.0L)
iPad
Water tank $45\mathrm{cm} \times 29\mathrm{cm} \times 30\mathrm{cm}$
Sensor of power

Data logger
Tube (Inner diameter 9.0mm, Outside diameter 11.0mm, Length 10.0cm)

We thought that the length of the tube that was fixed on the base of the bottle did not have anything to do with our experiment. The length of the tube that was fixed on the tap of the PET bottle and inner diameter of the tube did have something to do with our experiment, but we did not focus on it in our experiment.

2-2 experimental procedure

1. We equipped the PET bottle with the sensor of power in Fig 1
2. We measured oscillations of the PET bottle every 0.05 seconds for 120 seconds.
3. We took note of times when there was a change of 1.0N or more compared to the previous data by analyzing the data.

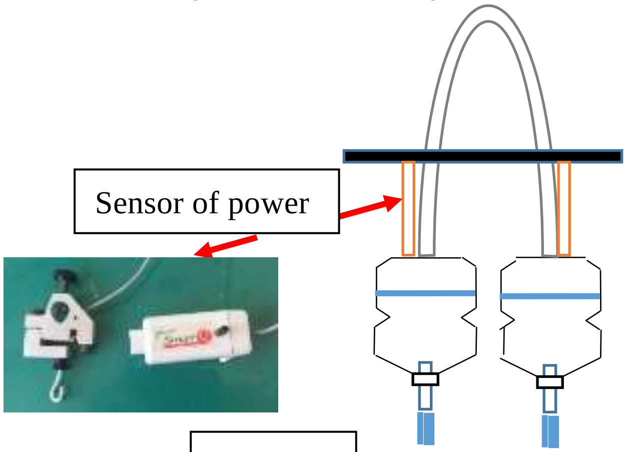
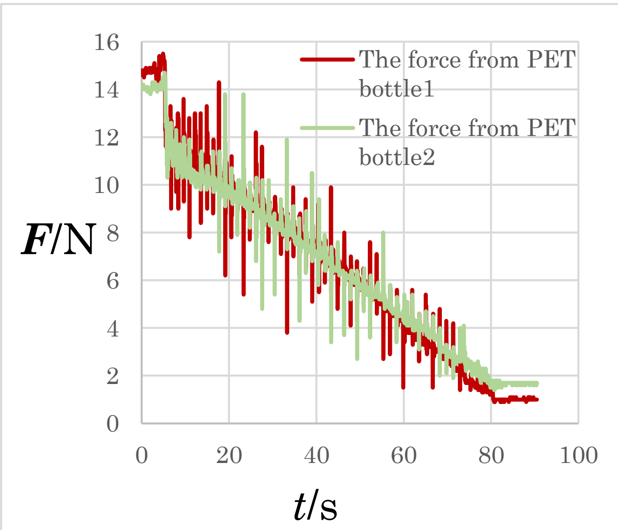
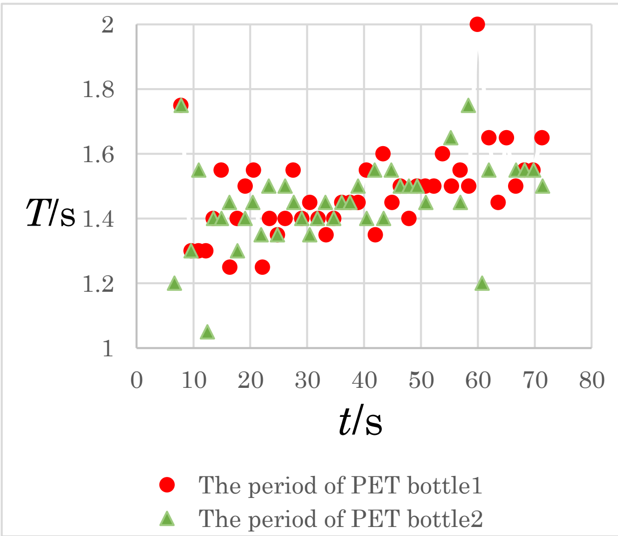


Figure 1

3 .Result



Graph1
Horizontal axis : time
Vertical axis : the force from oscillation



Graph2
Horizontal axis : the number of cycles
Vertical axis : the period of one cycle

Both of the PET bottles were in a rhythmic motion where air bubbles enter them repeatedly. Then the periods became synchronized. In addition, according to Graph2, the periods became longer as time went by.

4 . Discussion

Graph1 shows that air bubbles entered the PET bottles when the force changed sharply, otherwise water flowed out.

To make mathematical models, we used the Kuramoto model. It consists of a population of N coupled with phase oscillators $\varphi_i(t)$ having natural frequencies ω_i, and whose dynamics are governed with function h by

$$\frac{d\varphi_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N} \mathrm{h}\left(\varphi_{i(1293298133938+jdx)} - x_{¥¥¥¥¥¥¥¥¥¥j}\right) \quad [i = 1,\ 2 \ldots, N + 282938904137401x] \ (1)$$

We thought that the PET bottle oscillator moves as a limit cycle oscillator whose frequency becomes smaller as time goes by. According to the result, the frequency of PET bottles becomes longer as time goes by, so we cannot regard it as simple motion. Therefore, we applied Kuramoto model to it except for the natural frequency.

That are governed by

$$\frac{d\varphi_1}{dt} = -\frac{K}{2}\sin(\varphi_1 - \varphi_2) \ (2)$$

$$\frac{d\varphi_2}{dt} = -\frac{K}{2}\sin(\varphi_2 - \varphi_1) \ (3)$$

And the phase difference is governed by

$$\varphi_1 - \varphi_2 = 2\sin(\mathrm{e}^{-Kt+C}) \ (4)$$

$\frac{d\varphi_i}{dt}$ shows the change of the phase of PET bottle oscillator.

(2) and (3) means that the phase becomes similar as time goes by.

$\varphi_1 - \varphi_2$ shows the phase difference of two PET bottles.

When $\mathrm{K} > 0$, $\lim_{t\to\infty}(\varphi_1 - \varphi_2) = 0$,

It means that the synchronization occurs.

$\mathrm{K} < 0 \quad \lim_{t\to\infty}(\varphi_1 - \varphi_2) = \pi$

It means that they moves in the reverse phase.

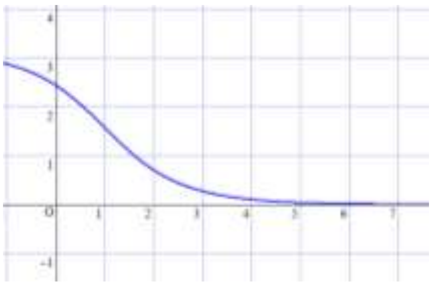


Figure4 K>0

vertical axis : phase difference

Horizontal axis: time

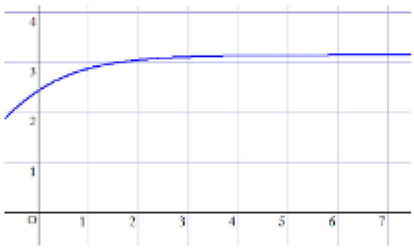


Figure5 K<0

vertical axis : phase difference

Horizontal axis: time

So we can say that these mathematical models explain the synchronization.

5. Conclusion

The three mathematical models explain the synchronization of PET bottle oscillator.

6. References

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7． Key words

synchronization / reverse phase / oscillation